PRIIMEK	IME	VPISNA ŠTEVILKA	SMER

NALOGA	TOČKE
1.	
2.	
3.	
4.	
SKUPAJ	

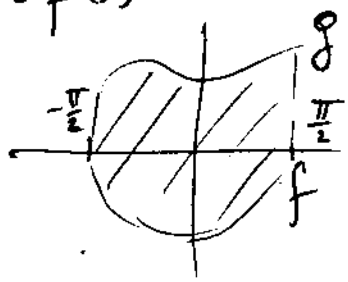
RAČUNSKI DEL IZPITA IZ MATEMATIČNE ANALIZE II (3.9.2007)

Navodilo: vsako nalogo rešuj na strani, kjer je napisana. Če bo naloga reševana kje drugje, mora biti to posebej označeno. Veliko uspeha pri reševanju!

1. naloga: Izračunaj geometrijski vztrajnostni moment glede na y-os nosilca s prerezom, ki ga omejujeta grafa $g(x) = e^{x^3}$ in $f(x) = -\cos x$ na intervalu $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Geometrijski vztrajnostni moment glede na x-os nosilca s prerezom, ki ga omejujeta grafa funkcij $g(y)$ in $f(y)$, kjer za vsak $y \in [a, b]$ velja $g(y) \geq f(y)$ je $I = \int_a^b y^2 (g(y) - f(y))\, dy$ (25 točk)

na $[-\frac{\pi}{2}, \frac{\pi}{2}]$: $g(x) = e^{x^3} > 0 \geq -\cos x = f(x)$



$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \left(e^{x^3} + \cos x\right) dx =$$

lih.f.   liha f.

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 e^{x^3} dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x\, dx = \frac{1}{3}\int_{-\frac{\pi^3}{8}}^{\frac{\pi^3}{8}} e^t dt + (x^2 \sin x)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \sin x\, dx =$$

$t = x^3$ $dt = 3x^2 dx$ $x^2 dx = \frac{dt}{3}$ $\alpha = -\frac{\pi^3}{8}$ $\beta = \frac{\pi^3}{8}$ | $u = x^2 \rightsquigarrow du = 2x\,dx$ $dv = \cos x\,dx \rightsquigarrow v = \sin x$

$$= \frac{1}{3}\left(e^t\right)\Big|_{-\frac{\pi^3}{8}}^{\frac{\pi^3}{8}} + \frac{\pi^2}{4} + \frac{\pi^2}{4} - 4\int_0^{\frac{\pi}{2}} x \sin x\, dx = \frac{1}{3}\left(e^{\frac{\pi^3}{8}} - e^{-\frac{\pi^3}{8}}\right) + \frac{\pi^2}{2} - 4\left(\overbrace{-x\cos x\Big|_0^{\frac{\pi}{2}}}^{0} + \int_0^{\frac{\pi}{2}} \cos x\, dx\right)$$

$u = x \rightsquigarrow du = dx$

$dv = \sin x\,dx \rightsquigarrow v = -\cos x$

$$= \frac{1}{3}\left(e^{\frac{\pi^3}{8}} - e^{-\frac{\pi^3}{8}}\right) + \frac{\pi^2}{2} - 4\left(\sin x\right)\Big|_0^{\frac{\pi}{2}} = \frac{1}{3}\left(e^{\frac{\pi^3}{8}} - e^{-\frac{\pi^3}{8}}\right) + \frac{\pi^2}{2} - 4$$

2. naloga: S pomočjo Taylorjeve formule izračunaj vrednost sin(0, 1) na štiri decimalna mesta natančno. (Rezultat pustite v obliki ulomka.) (25 točk)

$f(x) = \sin x \rightsquigarrow 0$

$f'(x) = \cos x \rightsquigarrow 1$

$f''(x) = -\sin x \rightsquigarrow 0$

$f'''(x) = -\cos x \rightsquigarrow -1$

$\vdots$

$$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + r_n$$

$$f^{(n)}(x) = \begin{cases} \pm \sin x & ; n\text{ - sodo št.} \\ \pm \cos x & ; n\text{ - liho št.} \end{cases}$$

$$\Rightarrow r_n = \frac{f^{(n)}(\theta x)}{(n+1)!} \cdot x^{n+1}$$

$$|r_n| \leqslant \frac{|x|^{n+1}}{(n+1)!}$$

želimo

za $x = 0{,}1 = \frac{1}{10}$: $|r_n| \leq \frac{1}{10^{n+1} \cdot (n+1)!} < 0{,}0001 = \frac{1}{10000}$

$\Downarrow$

$10000 < 10^n \cdot 10 \cdot (n+1)!$

$1000 < 10^n \cdot (n+1)!$

$\cdot n = 2$: $10^2 \cdot 3! = 600 < 1000$

$\boxed{n = 3}$ $10^3 \cdot 4! = 24000 > 1000 \leftarrow$ ustreza

$$f(x) = x - \frac{x^3}{3!}$$

$$\sin(0{,}1) = f\left(\tfrac{1}{10}\right) = \frac{1}{10} - \frac{1}{10^3 \cdot 3!} \doteq \underline{\underline{\frac{1}{10} - \frac{1}{6000}}}$$

3. naloga: Preveri ali za funkcijo $f(x,y) = x\Psi\left(\frac{x}{y}\right) + \Phi(xy)$, kjer sta Ψ in Φ zvezno odvedljivi funkciji, velja enakost:

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f.$$

(20 točk)

$$\frac{\partial f}{\partial x} = \Psi\left(\frac{x}{y}\right) + x \cdot \Psi'\left(\frac{x}{y}\right) \cdot \frac{1}{y} + \phi'(xy) \cdot y$$

$$= \Psi\left(\frac{x}{y}\right) + \frac{x}{y} \cdot \Psi'\left(\frac{x}{y}\right) + y \cdot \phi'(xy)$$

$$\frac{\partial f}{\partial y} = x \cdot \Psi'\left(\frac{x}{y}\right) \cdot \left(-\frac{x}{y^2}\right) + \phi'(xy) \cdot x$$

$$= -\frac{x^2}{y^2} \cdot \Psi'\left(\frac{x}{y}\right) + x \cdot \phi'(xy)$$

$$x \cdot \frac{\partial f}{\partial x} + y \cdot \frac{\partial f}{\partial y} = x \cdot \Psi\left(\frac{x}{y}\right) + \cancel{\frac{x^2}{y} \cdot \Psi'\left(\frac{x}{y}\right)} + xy\,\phi'(xy)$$

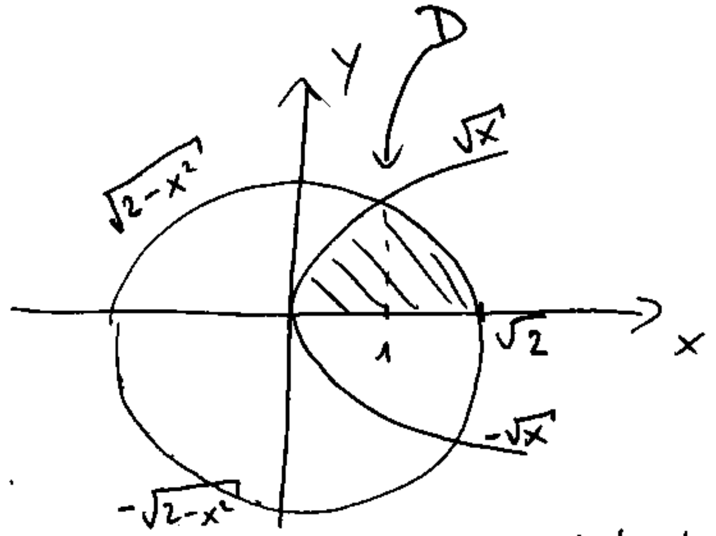
$$- \cancel{\frac{x^2}{y} \Psi'\left(\frac{x}{y}\right)} + xy\,\phi'(xy) =$$

$$= x \cdot \Psi\left(\frac{x}{y}\right) + 2xy\,\phi'(xy) \neq f$$

Enakost ne velja.

4. naloga: Izračunaj volumen telesa, ki ga omejujeta graf funkcije $f(x,y) = xy$ in ravnina $z = 0$ na območju $D = \{(x,y) \in \mathbb{R}^2 | x^2 + y^2 \leq 2, y^2 \leq x \text{ in } 0 \leq y\}$ (30 točk)

$$f(x,y) = \overset{x \geqslant 0}{x}\,\overset{y \geqslant 0}{y} \geqslant 0 \text{ na } D$$



presečišče obeh krivulj:

$x^2 + y^2 = 2 \quad , \quad x = y^2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\nexists\ \cancel{x=-2} \qquad \boxed{x = 1}$

$$V = \int_0^1 dx \int_0^{\sqrt{x}} xy\, dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} xy\, dy =$$

$$= \int_0^1 x \cdot \left(\frac{y^2}{2}\right)\Big|_0^{\sqrt{x}} dx + \int_1^{\sqrt{2}} x \cdot \left(\frac{y^2}{2}\right)\Big|_0^{\sqrt{2-x^2}} dx =$$

$$= \frac{1}{2}\int_0^1 x^2\, dx + \frac{1}{2}\int_1^{\sqrt{2}} \underbrace{x \cdot (2-x^2)}_{2x - x^3}\, dx = \frac{1}{2}\, \frac{x^3}{3}\Big|_0^1 + \frac{1}{2} \cdot \left(\cancel{2}\frac{x^2}{\cancel{2}} - \frac{x^4}{4}\right)\Big|_1^{\sqrt{2}} =$$

$$= \frac{1}{6} + \frac{1}{2} \cdot \left(\cancel{2} - \cancel{1} - \cancel{1} + \frac{1}{4}\right) = \frac{1}{6} + \frac{1}{8} = \frac{7}{24}$$